NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

THESIS

PROBABILITY MODELS FOR ASSESSING THE VALUE OF BATTLE DAMAGE ASSESSMENT IN THE DEFENSE AGAINST SEQUENTIAL THEATER MISSILE ATTACKS

by

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March, 1996

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REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE March 1996	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE: PROBABILITY MODELS FOR ASSESSING THE VALUE OF BATTLE DAMAGE ASSESSMENT IN THE DEFENSE AGAINST SEQUENTIAL THEATER MISSILE ATTACKS	5. FUNDING NUMBERS
6. AUTHOR(S): Shing-Jen Song	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE

13. ABSTRACT *(maximum 200 words)*

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14. SUBJECT TERMS Battle Damage Assessment(BDA); Theater Missile Defense; Measure of Effectiveness (MOE); Value of Information	15. NUMBER PAGES 59
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION ABSTRACT UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18 298-102

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE
IN
OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
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EXECUTIVE SUMMARY

In recent years, many countries have acquired short range ballistic missiles. As a result, many other countries have become possible targets for these missiles. Even when these missiles carry conventional warheads they levy a psychological and political penalty on the defending nation. Naturally, the defending nations are seeking effective ways to repulse ballistic missile attacks.

Defense, for most of the nations who believe they may be subject to a ballistic missile attack, is in the terminal stage of the ballistic missile trajectory; that is, when the missile reenters the atmosphere in a path towards its intended target. This thesis explores probability models to investigate the effects and value of information available to perform sequential tasks in the defense against a ballistic missile attack.

The information available for these sequential tasks encountered in a ballistic missile defense is termed battle damage assessment (BDA). This thesis investigates different levels of information for the battle damage assessment and the varying impacts this will have on the outcome of the battle. It is assumed that additional information could increase the effectiveness of the defense's weapons systems. Scenarios are explored where the enemy uses electronic warfare and decoys to disrupt the defender's acquisition of information. The intent of these models is to produce results for allocating limited resources and maximizing the available kill time and the probability of destroying a real missile rather than an electronic image or a decoy. The models are discussed from both the view point of the defender and the attacker.

First, the author presents a generic scenario which will be used throughout the rest of the study to analyze probabilities for both defender and attacker. Each scenario is further categorized by an investigation with battle damage assessment information and a scenario in which there is no battle damage information. Next, a measure of effectiveness is developed for the investigation. Finally, the author presents an investigation of the effect

of countermeasures on all the scenarios developed earlier in the study and a more detailed presentation of the effects of electronic warfare on the defensive systems.

The implication of the study are that theater missile defense would require enormous investments in research and the resources of the defending country. If the defender pursues a theater missile defense it must be done very carefully. When a successful attack takes place, even the crudest of ballistic missile might have a psychological effect out of proportion to the military value of the weapon. For example, in the Gulf War, the allies expended enormous resources to destroy missiles which when launched, rarely found a target and in any case, were not aimed at any militarily valuable target. These weapons had political effects well beyond their real military value.

Throughout the study, it is apparent that theater ballistic missile defense is closely tied to the battle damage assessment information system. Without the development of these systems it will be impossible for the defender to allocate the resources of kill time and weapons to achieve the best outcome for the battle.

The effect of electronic warfare and different levels of deception on the battle damage assessment information can have important effects and should be carefully investigated to ensure that the defender receives the benefit of more information. An improved kill rate on the first target in the scenarios presented, increases the value of battle damage assessment.

Suggested follow-on research is the problem of target identification to determine whether the target is a missile or a decoy. With reliable target identification information, the problems presented in this thesis would be reduced to improvements in weapons efficiency, and the scenarios themselves would consist of which target to identify first rather than which target is the most probable warhead.

This thesis presents a very simple scenario. However, the probability models identify some real world problems. The author found that the information provided by a battle damage assessment greatly affects the effectiveness of any defense scenario.

Sometimes this effect is unexpected, and this subject should be rigorously studied before any such system is actually built.

I. INTRODUCTION

A. BACKGROUND

Technological advances have made it possible for many countries to acquire ballistic missiles. As a result of the advances in technology and the willingness of some countries to sell that technology, the likelihood of ballistic missile attack is increasing. Ballistic missiles can deliver conventional and unconventional weapons over extended ranges. Even when a ballistic missile carries a conventional warhead it can still cause significant damage to the capability of the defender. From the experience of The Gulf War, we know that ballistic missile attacks can cause problems not only to the infrastructure and to human lives but can also create psychological and political problems in that a nation lives in constant fear of attack. The nation's economy will suffer and the nation's internal cohesiveness may decrease.

Many countries are seeking an effective way to defend themselves against ballistic missile attacks. For example, the United States started the Ballistic Missile Defense Program (BMD) in April 1984 [Ref. 1]. The current BMD program contains three major parts:

1. Theater Missile Defense

Theater Missile Defense (TMD) program is intended to provide highly effective TMDs to forward deployed forces and to U.S. Forces and allies. The TMD defense can take many forms. The Joint Chief of Staff categorizes TMD mission needs as, first, a passive defense to enhance the survivability of friendly forces and assets; second, a battle Management/Command, Control and Communications and Intelligence system (BM/C3I) to provide effective communications, command and control of the TMD operation and to ensure data flow; third, an attack operations (Counter-force) for the destruction of the enemy's capability to launch missiles; and finally an active defense to intercept the Theater Ballistic Missile (TBM) in flight so as to either destroy the TBM or negate the effects of the warhead.

2. U. S. National Missile Defense

National Missile Defense is a research and development project for the development of ground based-defenses to protect the U.S. from a limited ballistic missile attack. The Army's National Missile Defense (NMD) system will operate with external Early Warning (EW) sensors (Space and Missile Tracking System, DSP and EWR) and the United States Space Command's (USSPACECOM) Command and Control Center via a Command-Level Battle Management Command Control and Communications (BMC3) network. The Army configuration of the proposed NMD system includes ground-based exo-atmospheric hit-to-kill interceptors, a ground-based phased array and national defense radar (for surveillance, tracking, object classification and kill assessment) and Battalion BMC3 (Bn BMC3) (for human-in-control, engagement planning, top level decision making and system communications) [Ref. 2].

3. Follow-On Research

Follow-on research supports more advanced BMD technologies. For example, to increase the effectiveness of the weapon system, issues related to the flow and utilization of information play an important role in the BMD program. The BMD program represents an investment of $34,683.3 million dollars [Ref. 1]. These resources can be used to improve the effectiveness of weapon systems or improve the acquisition flow and utilization of information. However, how do we evaluate the value of information? What are the trade-offs of investing money on weapon or information? How can we effectively use information to get the best results?

B. ACTIVE DEFENSE

Considering the trajectory of a ballistic missile, the active defense can be divided into three opportunity stages:

1. The Boost Stage

The boost stage is the early portion of missile flight. In this stage the missile engine will burn to produce thrust until it reaches terminal velocity. There are advantages to engaging in this phase. The first is that the missile is in its early flight and cannot expel its multiple warheads and decoys. The second is that the missile is probably still above the attackers' territory and will not cause any damage to the defender.

Because of short time and longer distances represented by this stage, the stage requires a relatively higher technology to intercept a missile. We may use space-based sensors and weapons ("Brilliant Eyes/Pebbles") to accomplish the task. For the reason of defense budget resources, this method would probably not be used in the post Cold-War world.

2. The Midcourse Stage

In this stage the missile follows a ballistic path. It is desirable to intercept in this stage because of the advantage of destroying the missile outside the defender territory or at very high altitude. At this stage it is less difficult to detect a target and to guide an anti-ballistic missile weapon than in the boost stage. Additionally, the missile is moving relatively slower than in the terminal stage. However, the enemy may use decoys or other countermeasures to make the defenders' task more difficult.

3. The Terminal Stage

In this stage the missile re-enters the atmosphere and follows a downward path to its target area. The defender at this stage has more resources to destroy the missile. For example, the use of air-defense missiles and aircraft. The problem at this stage is that the missile is moving at a very high speed and offers only a small window for interception. Additionally, destruction of the missile will result in scattering debris that may still result in the missile being an effective political weapon even though a less then effective military one.

4. Summary

The stages of the ballistic missile trajectory can be thought of as three unique opportunities or time windows for the defense to engage the missile attack.

C. THE ROLE OF INFORMATION

Information is power in modern warfare and modern business. An effective system must combine information technology with other resources to achieve the best results. The

Ballistic Missile Defense needs an extremely accurate and fast strike capability which can only be accomplished with a high speed command, control, communication, intelligence (C3I) and battle awareness information system. Information can be obtained from external sources, such as satellites, early-warning air radar, ground-based radar and other sensor systems. The system may produce information on target identification, detection, tracking, and project battle damage. Different levels of information will have different impacts on the outcome of the battle. Additional information could increase the effectiveness of the defensive weapon system. On the other hand, the enemy could use deception techniques on the information-gathering (EW and Decoy measures) methods to disrupt the acquisition of information which would decrease the effectiveness of defensive weapons.

This thesis seeks to use probability models to investigate the effects and value of information availability on sequential tasks encountered in defense against missile attacks.

II. MODEL DESCRIPTION

A. THE GENERIC SCENARIO

This thesis is mainly based on study of a generic scenario and the assumptions in the following paragraphs. These assumptions restrict the problem to two tasks and the decision as to which task to pursue to obtain maximum combat effectiveness. The tasks are performed by a weapon system (server). The weapon system (server) contains a sensor-C3 sub-system and a ballistic missile interceptor. It is confronted with the opportunity to address particular and temporarily available tasks; that is to destroy incoming targets. Each task requires an acquisition time T_a to acquire enough detection and identification information for an incoming missile. While the tracking information is available the interceptor can start to deliver a lethal kill. After a lethal kill time T_k the target will be destroyed.

Because of the ballistic missile trajectory there is only a limited period of time (window) to engage target i, i.e. accomplish Task i. Let's assume that the server starts to work on Task 1 from time $t_{11} = 0$ and the time available to complete Task 1 extends to $t_{12} > t_{11}$; The opportunity to work on Task 2 begins at t_{21}, where $t_{21} \geq 0$, resulting in an overlapped region between the two tasks of $t_{12} - t_{21}$. (See Figure 2-1); the opportunity to

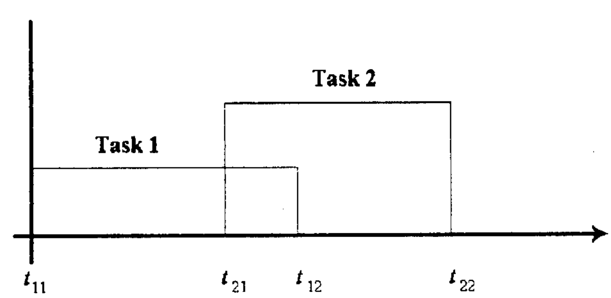

Figure 2-1. Overlapping Tasks

work on Task 2 ends at time t_{22}.

More than two tasks could be overlapping in a real world problem, but for the purposes of this study only two tasks will be considered. Assume that at any moment the time needed to complete service on Task i is $T_i = T_a + T_k$. For a glimpse-type sensor the acquisition time T_a will be modeled as an exponential random variable which has mean $1/\lambda_a$. Depending on the characteristics of the weapon, we may choose different distributions for the lethal kill time T_k. For convenience, we assume that the time to complete task i has an exponential distribution with mean $1/\lambda_i$. Since $1/\lambda_i$ is the expected survival time of target i, the parameter λ_i can be thought of as the relative efficiency factor of an anti-ballistic missile weapon. A weapon with a higher λ_i is a more efficient weapon, because within a limited time interval the chance to complete a task is higher.

Assume that at any moment in time there is a chance to kill a target that is independent of events that have preceeded that point (t, t + dt) in time (= λ_i dt, where dt is a small time, although the completion rate λ_i could depend on time and distance: $\lambda_i(t, d)$. We will allow the server, shooter, or sensor to "attack" just one task at a time. However, under some conditions one could co-allocate "shots" across several tasks or targets. A BMD system will assign different values to different targets. If we finish a task, we will be rewarded with the task's value. On the other hand if we do not finish then the task will not have any value. The task value relates to the damage the missile might cause to the defender in case it is not intercepted.

The issue of primary interest in this paper is how to best allocate limited server capacity to the two tasks in the light of the available information. Ultimately, this will determine how much effort to expend for an increase in opportunity time. This will also lower service (reaction/response/kill) times of the server and reduce the effect of target duping and other attacker tactics.

The following discussion of models reflects varying levels of information that might be available to the defender or server. It is assumed that the server has knowledge of the various times t_{21}, t_{12}, t_{22}, sufficiently in advance to allow the server to accommodate the decision making process.

B. MODEL WITH PERFECT ACQUISITION INFORMATION BUT WITHOUT BATTLE DAMAGE ASSESSMENT (BDA) INFORMATION

In this model we will assume that the BMD system has perfect target acquisition information, including detection, identification, and localization information. So there is no acquisition time model required. The defense system knows when Task *i* is available but does not know when it is or will be completed. Since there is no-BDA information the system will use a threshold policy to engage a target. The rule of engagement is first-come, first-served. The system starts immediately with Task 1 and pursues service of Task 1 for threshold time τ_{12} (here $t_{21} < \tau_{12} < t_{12}$). Then the system switches to Task 2 after τ_{12} and continues to t_{22}. If we finish Task 1 we will have received the value for the first target, V_1. If the system completes Task 2 in time the result is value V_2. Assume the task values are additive. Realistically, in a real world situation, the values need not add; including the defended target's value accounting for the effect of a second strike on the same target may be smaller in terms of infrastructure or more in terms of personnel.

1. Expected Reward

We will assess the *reward* of a policy by the expected *sum* of the values of the two tasks accomplished. Let $V(\tau_{12})$ be the expected reward from following the above *rule*; that is, if the system switches targets between end of the available time for Task 1, and the beginning of the available time for Task 2:

$$\begin{aligned} V(\tau_{12}) &= V_1\left[1-e^{-\lambda_1\tau_{12}}\right]+V_2\left[1-e^{-\lambda_2(t_{22}-\tau_{12})}\right] \\ &= V_1+V_2-\left[V_1e^{-\lambda_1\tau_{12}}+V_2e^{-\lambda_2(t_{22}-\tau_{12})}\right] \text{ if } t_{21}\le\tau_{12}\le t_{12}\ ; \end{aligned} \qquad \textbf{(1)}$$

cf. Gaver and Jacobs (1996) [Ref. 7].

The *reward* if the system switches tasks at beginning of the available time for Task 2 (τ_{12} = t_{21})

$$V(t_{21}) = V_1\left(1-e^{-\lambda_1 t_{21}}\right) + V_2\left(1-e^{-\lambda_2(t_{22}-t_{21})}\right)$$
$$= V_1 + V_2 - \left[V_1 e^{-\lambda_1 t_{21}} + V_2 e^{-\lambda_2(t_{22}-t_{21})}\right]. \tag{2}$$

The *reward* if the system switches at the end of the available time for Task 1 ($\tau_{12} = t_{12}$)

$$V(t_{12}) = V_1\left(1-e^{-\lambda_1 t_{12}}\right) + V_2\left(1-e^{-\lambda_2(t_{22}-t_{12})}\right)$$
$$= V_1 + V_2 - \left[V_1 e^{-\lambda_1 t_{12}} + V_2 e^{-\lambda_2(t_{22}-t_{12})}\right]. \tag{3}$$

In fact, it is convenient to study the decrement to achieving the maximum reward $V_1 + V_2$ on a fractional basis; this is

$$D(\tau_{12}) = 1 - \frac{V(\tau_{12})}{V_1 + V_2}$$
$$= \left[r_1 e^{-\lambda_1 \tau_{12}} + r_2 e^{-\lambda_2(t_{22}-\tau_{12})}\right], \quad t_{21} \le \tau_{12} \le t_{12} \tag{4}$$

where $\quad r_1 = V_1/(V_1+V_2),\; r_2 = V_2/(V_1+V_2).$

We want to pick $\tau_{12} = \tau_{opt}$ that minimizes this decrement. Now

$$\frac{\partial D(\tau_{12})}{\partial \tau_{12}} = \left[-r_1\lambda_1 e^{-\lambda_1 \tau_{12}} + r_2\lambda_2 e^{-\lambda_2(t_{22}-\tau_{12})}\right]$$
$$\frac{\partial^2 D(\tau_{12})}{\partial^2 \tau_{12}} = \left[r_1\lambda_1^2 e^{-\lambda_1 \tau_{12}} + r_2\lambda_2^2 e^{-\lambda_2(t_{22}-\tau_{12})}\right] > 0. \tag{5}$$

This tells us that $D(\tau_{12})$ is *bowl-shaped,* having just one bottom as shown in Figure 2-2. One of the above pictures must describe the situation; which one depends on the various parameters. In case (I), the unrestricted minimum of $D(\tau_{12})$ occurs before t_{21}, the time when Task 2 appears. A change to Task 2 at this time cannot be sensible under the current model because Task 2 is not available; the best feasible time must be at t_{21} for this case. If case (II) prevails, then there is a bona-fide time *between* t_{21} and t_{12} to change to Task 2. If case (III) holds, then it is best to continue with Task 1 to the end ("a bird in hand…") and then switch over to Task 2. Note that only one switch is allowed.

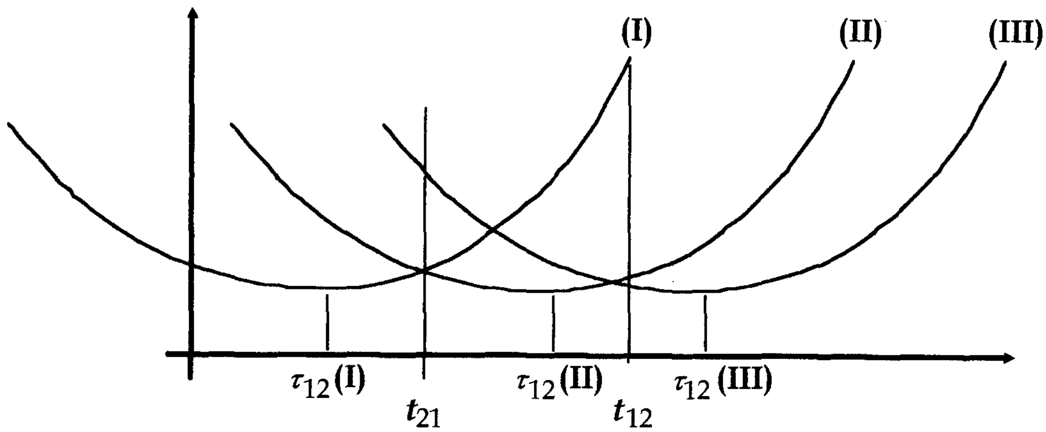

Figure 2-2. Bowl-Shaped

The above can be formalized by finding $\widetilde{\tau}_{12}$, the global/unrestricted minimizing value of changeover time threshold by minimizing (finding bowl-bottom coordinate) of $D(\tau_{12})$. Then

$$
\begin{aligned}
\tau_{\text{opt}} &= t_{21} \quad \text{if } \widetilde{\tau}_{12} \le t_{21} && \textit{CASE I} \\
&= \widetilde{\tau}_{12} \quad \text{if } t_{21} \le \widetilde{\tau}_{12} \le t_{12} && \textit{CASE II} \\
&= t_{12} \quad \text{if } \widetilde{\tau}_{12} \ge t_{12} && \textit{CASE III}
\end{aligned}
\tag{6}
$$

To find a formula for $\widetilde{\tau}_{12}$ differentiate $D(\tau_{12})$ and solve $\dfrac{\partial D}{\partial \tau_{12}} = 0$.

$$
\begin{aligned}
(V_1 + V_2)\frac{\partial D(\tau_{12})}{\partial \tau_{12}} &= -\frac{\partial V(\tau_{12})}{\partial \tau_{12}} = -V_1\lambda_1 e^{-\lambda_1 \tau_{12}} + V_2\lambda_2 e^{-\lambda_2(t_{22}-\tau_{12})} \\
(V_1 + V_2)\frac{\partial^2 D(\tau_{12})}{\partial^2 \tau_{12}} &= -\frac{\partial^2 V(\tau_{12})}{\partial^2 \tau_{12}} = +V_1\lambda_1^2 e^{-\lambda_1 \tau_{12}} + V_2\lambda_2^2 e^{-\lambda_2(t_{22}-\tau_{12})} > 0.
\end{aligned}
\tag{7}
$$

So, equating the first derivative to zero and solving gives:

$$e^{-(\lambda_1-\lambda_2)\tau_{12}} = \frac{V_2\lambda_2}{V_1\lambda_1}e^{-\lambda_2 t_{22}}$$

$$(\lambda_1+\lambda_2)\tau_{12} = \ln\left(\frac{V_1\lambda_1}{V_2\lambda_2}\right)+\lambda_2 t_{22} \tag{8}$$

$$\tau_{12} = \frac{1}{\lambda_1+\lambda_2}\left\{\ln\left(\frac{V_1\lambda_1}{V_2\lambda_2}\right)+\lambda_2 t_{22}\right\} \tag{9}$$

Case (I) holds if $\tau_{12} < t_{21}$, in which case $\tau_{opt} = t_{21}$.

Case (II) holds if $t_{21} < \tau_{12} < t_{12}$; then $\tau_{opt} = \tau_{12}$.

Case (III) holds if $\tau_{12} > t_{12}$; then $\tau_{opt} = t_{12}$;

cf. Gaver and Jacobs (1996) [Ref. 7].

For Case (I) where τ_{12} must be less then t_{21} in the solution to Equation 9 we have

$$\tau_{12} = \frac{1}{\lambda_1+\lambda_2}\left(\ln\left(\frac{V_1\lambda_1}{V_2\lambda_2}\right)+\lambda_2 t_{22}\right) \le t_{21}$$

$$\ln\left(\frac{V_1\lambda_1}{V_2\lambda_2}\right) \le t_{21}(\lambda_1+\lambda_2)-\lambda_2 t_{22} \tag{10}$$

$$V_1 \le \frac{\lambda_2 V_2}{\lambda_1}e^{(\lambda_1+\lambda_2)t_{21}-\lambda_2 t_{22}}$$

then τ_{12} will less than t_{21}, in which case the server will choose $\tau_{opt} = t_{21}$.

Case(II) if $t_{21} < \tau_{12} < t_{12}$; then $\tau_{opt} = \tau_{12}$.

Case(III) if $V_1 \ge \frac{\lambda_2 V_2}{\lambda_1} v_2 e^{\lambda_1 t_{22}}$; then $\tau_{12} > t_{12}$, in which case $\tau_{opt} = t_{12}$;

From Equation 9 we know that τ_{opt} is governed by ln ($V_1\lambda_1 / V_2\lambda_2$). If $V_1\lambda_1 < V_2\lambda_2$ then ln ($V_1\lambda_1$ / $V_2\lambda_2$) is negative. Notice that $V_1\lambda_1 < V_2\lambda_2$ is equivalent to $V_1 /(1/\lambda_1) = V_1/E(T_1) < V_2/E(T_2) = (V_1 /\lambda_2)$. This means when the expected reward value gained from killing Task 1 per unit time is less than Task 2; the system will allocate less kill time for Task 1 in order to maximize the total expected reward value. Under this situation the best strategy for the server to get more expected return value is to switch early to work on Task 2. On the other hand if the expected return value per unit time of Task 1 is more than Task 2, then the server should spend more time on Task 1. This property indicates that a system will spend more time on a high value and high kill rate target, which is intuitive.

For example, suppose two tasks arrive simultaneously and the available engagement time is one unit for both tasks; the Task 2 has a fixed value 5; λ_1 and λ_2 each vary from 0.05 to 5. Figures 2-3a to 2-3d show that if the Task 1 is more important (higher value) than Task 2, then the system will assign relatively more value, i.e. larger τ_{12} to Task 1. The model reflects the fact that the best strategy for the system to get more return value is to spend more time on Task 1.

In Figure 2-3a Task 1 value (V_1=1) is less than Task 2 value (V_2=5). In order to get more value the system will switch to Task 2 early. So the threshold (Opt_tau12) is less than 0.5 in most cases. However, the right-inner part (area A) of the figure shows that if Task2 is relatively much harder to complete (i.e. λ_2 is very small) than Task1, we had better continue to work on the easier job instead of working on a job which we may not finish. In Figure 2-3b (V_1=V_2=5) for most of the cases we switch around 0.5 unit of time. However, the left-inner part (area B) shows that if λ_1 is very small the strategy to get more value is to switch early.

Figures 2-3c and 2-3d display results for a case in which V_1 is much higher than V_2. Most of the time the optimal τ_{12} is above 0.5 unit time. Notice that in Figure 3d area D is lower than it's vicinity. From Eq-9, we know that when λ_1 is relatively large then the optimal τ_{12} will decrease by the amount about proportional to $1/(\lambda_1+\lambda_2)$. This property indicates that when the system is efficient enough for the first target, the system should reserve some time for the second target.

Figure 2-4 displays results that when Task 2 value increases from 1 to 45 the optimal threshold value decreases. Figure 4d shows that the value of Task 2 (V_2= 45) is so great that the system had better switch over to Task 2 as soon as possible.

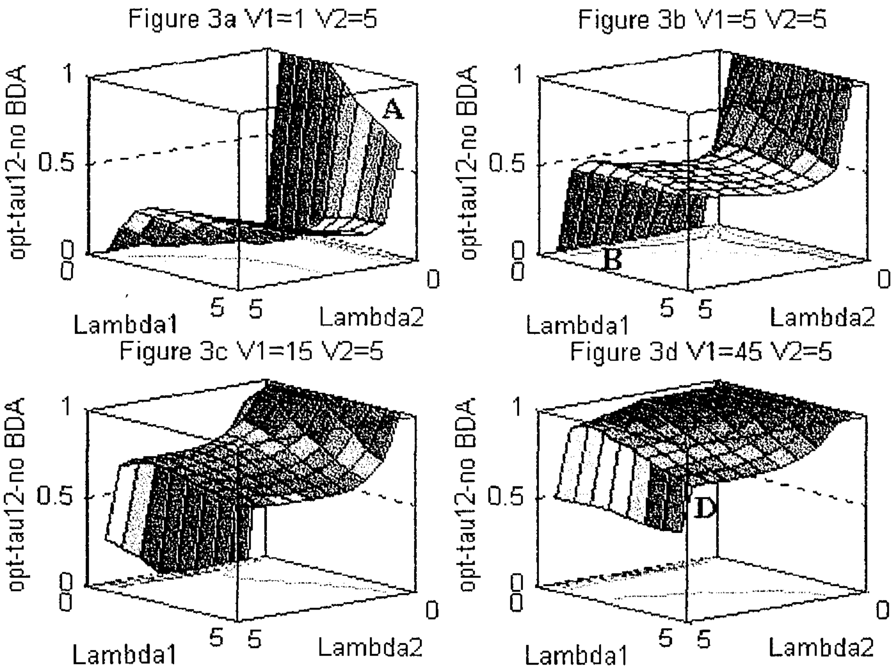

Figure 2-3. Optimal τ_{12} Increases As V_1 Increases

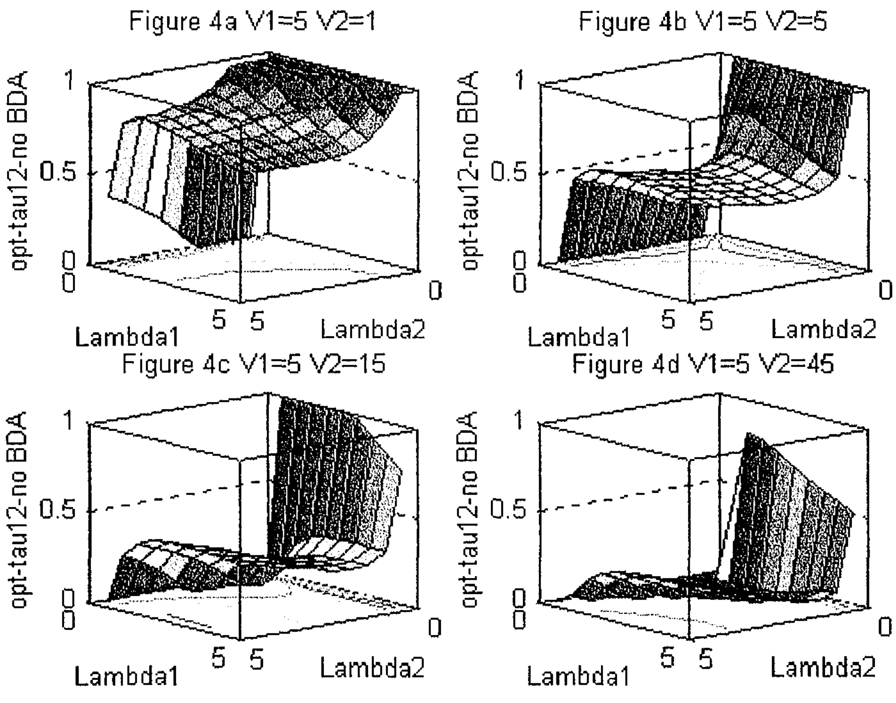

Figure 2-4. Optimal τ_{12} Decreases As V_2 Increases

However, when λ_2 increases to 5 the weapon efficiency for Task 2 become high enough so that the system will reserve some kill time for Task 1.

C. MODEL WITH PERFECT BDA INFORMATION

In this model, we assume that we have damage assessment information from a friendly source. The damage assessment information is knowledge that the target is killed when it is killed; that is knowledge that the task is completed when it is completed. The task values are the same as in the previous model. However, the rule of engagement is slightly different: the system starts immediately with Task 1 and pursues it for a (random) time min(T_1, τ_{12}); T_1 is the time to completion (acknowledgment of BDA) of Task 1 and τ_{12} is a threshold value. After the time, minimum (T_1, τ_{12}), the system switches to Task 2. We derive the expected return as follows:

By conditioning on $\boldsymbol{T}_1 = t_1$ [Ref. 3], the conditional expected return is

$$\begin{aligned} V(\tau_{12};t_1) &= V_1 + V_2\left(1-e^{-\lambda_2(t_{22}-t_{21})}\right), && t_1 \le t_{21} \\ &= V_1 + V_2\left(1-e^{-\lambda_2(t_{22}-t_1)}\right), && t_{21} < t_1 \le \tau_{12} \le t_{12} \\ &= V_2\left(1-e^{-\lambda_2(t_{22}-\tau_{12})}\right), && t_{21} \le \tau_{12} < t_1 . \end{aligned} \tag{11}$$

Now remove the condition:

$$\begin{aligned} V(\tau_{12}) &= V_1\left(1-e^{-\lambda_1 t_{21}}\right) + V_2\left(1-e^{-\lambda_1 t_{21}}\right)\left(1-e^{-\lambda_2(t_{22}-t_{21})}\right) \\ &\quad + V_1\left[\left(1-e^{-\lambda_1\tau_{12}}\right)-\left(1-e^{-\lambda_1 t_{21}}\right)\right] + V_2\int_{t_{21}}^{\tau_{12}}\left(1-e^{-\lambda_2(t_{22}-x)}\right)e^{-\lambda_1 x}\lambda_1 dx \\ &\quad + V_2\left[1-e^{-\lambda_2(t_{22}-\tau_{12})}\right]e^{-\lambda_1\tau_{12}} \\ &= V_1\left(1-e^{-\lambda_1\tau_{12}}\right) + V_2\Bigg[\left(1-e^{-\lambda_1 t_{21}}\right)\left(1-e^{-\lambda_2(t_{22}-t_{21})}\right) \\ &\quad + \left(1-e^{-\lambda_2(t_{22}-\tau_{12})}\right)e^{-\lambda_1\tau_{12}} + \left(e^{-\lambda_1 t_{21}}-e^{-\lambda_1\tau_{12}}\right) \\ &\quad - e^{-\lambda_1 t_{21}}e^{-\lambda_2(t_{22}-t_{21})}\frac{\lambda_1}{\lambda_1-\lambda_2}\left[1-e^{-(\lambda_1-\lambda_2)[\tau_{12}-t_{21}]}\right]\Bigg] \ ; \end{aligned} \tag{12}$$

cf. Gaver and Jacobs (1996) [Ref. 7].

If $\lambda_1 = \lambda_2 = \lambda$, then (12) becomes

$$V(\tau_{12}) = V_1\left(1 - e^{-\lambda\tau_{12}}\right) + V_2\left[1 - e^{-\lambda(t_{22}-t_{21})} - e^{-\lambda t_{22}}\lambda(\tau_{12} - t_{21})\right] \quad \textbf{(13)}$$

1. **Special Case: $\lambda_1 = \lambda_2 = \lambda$**

Assume $\lambda_1 = \lambda_2 = \lambda$. Note that a decision concerning the value of τ_{12} is only needed if $T_1 > t_{21}$. Hence, to determine the value of τ_{12}, we consider

$$\begin{aligned} f(\tau_{12}) &= E\left[V(\tau_{12}) \middle| T_1 > t_{21}\right] \\ &= V_1\left(1 - e^{-\lambda(\tau_{12}-t_{21})}\right) + V_2\left[1 - \left[e^{-\lambda(\tau_{12}-t_{21})} + e^{-\lambda(\tau_{12}-t_{21})}\lambda(\tau_{12} - t_{21})\right] e^{-\lambda(t_{22}-\tau_{12})}\right] \\ &= V_1\left(1 - e^{-\lambda(\tau_{12}-t_{21})}\right) + V_2\left[1 - e^{-\lambda(t_{22}-t_{21})} - e^{-\lambda(t_{22}-t_{21})}\lambda(\tau_{12} - t_{21})\right] \end{aligned} \quad \textbf{(14)}$$

for $t_{21} \le \tau_{12} \le t_{12}$.

If $V_1 \ge V_2$, the maximizing value of τ_{12} is t_{12} since the server is assumed not to return to Task 1.

Assume $V_1 < V_2$; note that $f(t_{12}) = V_2\left[1 - e^{-\lambda(t_{22}-t_{21})}\right]$.

Thus

$$f(\tau_{12}) - f(t_{21}) = V_1\left(1 - e^{-\lambda(\tau_{12}-t_{21})}\right) - V_2\, e^{-\lambda(t_{22}-t_{21})}\lambda(\tau_{12} - t_{21}). \quad \textbf{(15)}$$

Thus $f(\tau_{12}) - f(t_{12}) > 0$ if

$$f_\ell(\tau_{12}) \equiv V_1\left(1 - e^{-\lambda(\tau_{12}-t_{21})}\right) > V_2\, e^{-\lambda(t_{22}-t_{21})}\lambda(\tau_{12} - t_{21}) \equiv f_r(\tau_{21}). \quad \textbf{(16)}$$

Note that

$$\begin{aligned} f_\ell(t_{21}) &= f_r(t_{21}) = 0; \\ \frac{\partial}{\partial \tau_{12}} f_\ell(\tau_{12}) &= V_1 \lambda e^{-\lambda(\tau_{12}-t_{21})}; \end{aligned} \quad \textbf{(17)}$$

and

$$\frac{\partial}{\partial \tau_{12}} f_r(\tau_{12}) = V_2 \lambda e^{-\lambda(t_{22}-t_{21})}. \quad \textbf{(18)}$$

Thus, the maximizing τ_{12} equals t_{21} if $V_1 < V_2 e^{-\lambda(t_{22}-t_{21})}$.

Assume $V_2 > V_1 > V_2 e^{-\lambda(t_{22}-t_{21})}$. Note that

$$\frac{\partial}{\partial \tau_{12}}\left[f(\tau_{12}) - f(t_{21})\right] = V_1 \lambda e^{-\lambda(\tau_{12}-t_{21})} - V_2 \lambda e^{-\lambda(t_{22}-t_{21})} \quad \textbf{(19)}$$

and

$$\frac{\partial^2}{\partial^2 \tau_{12}}\left[f(\tau_{12}) - f(t_{21})\right] = -V_1 \lambda^2 e^{-\lambda(\tau_{12}-t_{21})} < 0\,. \quad \textbf{(20)}$$

Thus, if $V_2 > V_1 > V_2 e^{-\lambda(t_{22}-t_{21})}$, then the maximizing value of τ_{12} is

$$\tau_{12}^* = t_{22} - \frac{1}{\lambda}\log\left(\frac{V_2}{V_1}\right). \quad \textbf{(21)}$$

2. Summary

If $\lambda_1 = \lambda_2 = \lambda$, then the maximizing value of τ_{12} is as follows, cf. Gaver and Jacobs (1996) [Ref. 7].

$$\tau_{12}^* = \begin{cases} t_{21} & \text{if } V_1 \le V_2 e^{-\lambda(t_{22}-t_{21})} \\ t_{22} - \frac{1}{\lambda}\log\left(\frac{V_2}{V_1}\right) & \text{if } V_2 e^{-\lambda(t_{22}-t_{21})} < V_1 < V_2 e^{-\lambda(t_{22}-t_{12})} \\ t_{12} & \text{if } V_1 \ge V_2 e^{-\lambda(t_{22}-t_{12})} \end{cases} \quad \textbf{(22)}$$

3. General Case

Assume $\lambda_1 = (1 + c)\lambda$ for $c > -1$ and $\lambda_2 = \lambda$. Since the times to complete the tasks have independent exponential distributions, the lack of memory property suggests that we may assume $t_{21} = 0$ to find the maximizing τ_{12}. Rewriting (12), for $0 \le \tau_{12} < t_{12}$

$$\begin{aligned} f(\tau_{12}) &= V_1\left(1 - e^{-\lambda_1 \tau_{12}}\right) + V_2\left[1 - e^{-\lambda_2 t_{22}}\left[e^{-[\lambda_1-\lambda_2]\tau_{12}} + \frac{\lambda_1}{\lambda_1 - \lambda_2}\left[1 - e^{-[\lambda_1-\lambda_2]\tau_{12}}\right]\right]\right] \\ &= V_1\left(1 - e^{-\lambda(1+c)\tau_{12}}\right) + V_2\left[1 - e^{-\lambda t_{22}}\left[e^{-c\lambda\tau_{12}} + \frac{(1+c)}{c}\left[1 - e^{-c\lambda\tau_{12}}\right]\right]\right]. \end{aligned} \quad \textbf{(23)}$$

Thus,

$$f(\tau_{12}) - f(0) = V_1\left(1 - e^{-\lambda(1+c)\tau_{12}}\right) + V_2\left[e^{-\lambda t_{22}}\left[1 - e^{-c\lambda\tau_{12}} - \frac{(1+c)}{c}\left[1 - e^{-c\lambda\tau_{12}}\right]\right]\right]. \quad \textbf{(24)}$$

Further,

$$\frac{\partial}{\partial \tau_{12}}\left[f(\tau_{12}) - f(0)\right] = V_1 \lambda (1+c) e^{-\lambda(1+c)\tau_{12}} + V_2 e^{-\lambda t_{22}} \left[c\lambda e^{-c\lambda \tau_{12}} - \frac{(1+c)}{c} c\lambda e^{-c\lambda \tau_{12}} \right]. \quad \textbf{(25)}$$

Setting

$$\frac{\partial}{\partial \tau_{12}}\left[f(\tau_{12}) - f(0)\right] = 0 \quad \textbf{(26)}$$

results in the equation

$$V_1(1+c)e^{-\lambda \tau_{12}} = V_2 e^{-\lambda t_{22}}. \quad \textbf{(27)}$$

Solving for τ_{12} results in

$$\tau_{12}^0 = t_{22} - \frac{1}{\lambda}\log\frac{V_2}{V_1(1+c)}. \quad \textbf{(28)}$$

Thus, (12) with $t_{21} \geq 0$, $\lambda_1 = \lambda(1 + c)$, and $\lambda_2 = \lambda$, the maximizing τ_{12} is

$$\tau_{12}^* = t_{21} + \begin{cases} 0 & \text{if } V_1 < \dfrac{V_2 e^{-\lambda(t_{22}-t_{21})}}{1+c} \\ (t_{22} - t_{21}) - \dfrac{1}{\lambda}\log\dfrac{V_2}{V_1(1+c)} & \text{if } \dfrac{V_2}{1+c} e^{-\lambda(t_{22}-t_{21})} < V_1 < \dfrac{V_2 e^{-\lambda(t_{22}-t_{12})}}{1+c} \\ (t_{12} - t_{21}) & \text{if } \dfrac{V_2 e^{-\lambda(t_{22}-t_{12})}}{1+c} \leq V_1 \end{cases}$$

(29)

cf. Gaver and Jacobs (1996) [Ref. 7].

Comparing (29) and (10), with the parameterization $\lambda_1 = \lambda(1 + c)$, $\lambda_2 = \lambda$ for $c > -1$, note that with no-BDA information, the switching time $\tilde{\tau}_{opt} \equiv \tilde{\tau}_{12}$,

$$\tilde{\tau}_{opt} = \frac{1}{1+c}\tau_{12}^0 \quad \textbf{(30)}$$

with $\widetilde{\tau}_{12} = \tau_{12}^{0}$ only if $c = 0$. *Thus the maximum amount of time to be devoted to Task 1 after t_{21} is smaller if the decision maker has no-BDA information than if he has perfect BDA information.* The following graphs Figure 2-5 and Figure 2-6 use the same parameters as Figure 2-3 and Figure 2-4. The results display the property that no matter how the parameters change that the optimal threshold τ_{12} for BDA system tends to be greater than or equal the optimal threshold τ_{12} for no-BDA system.

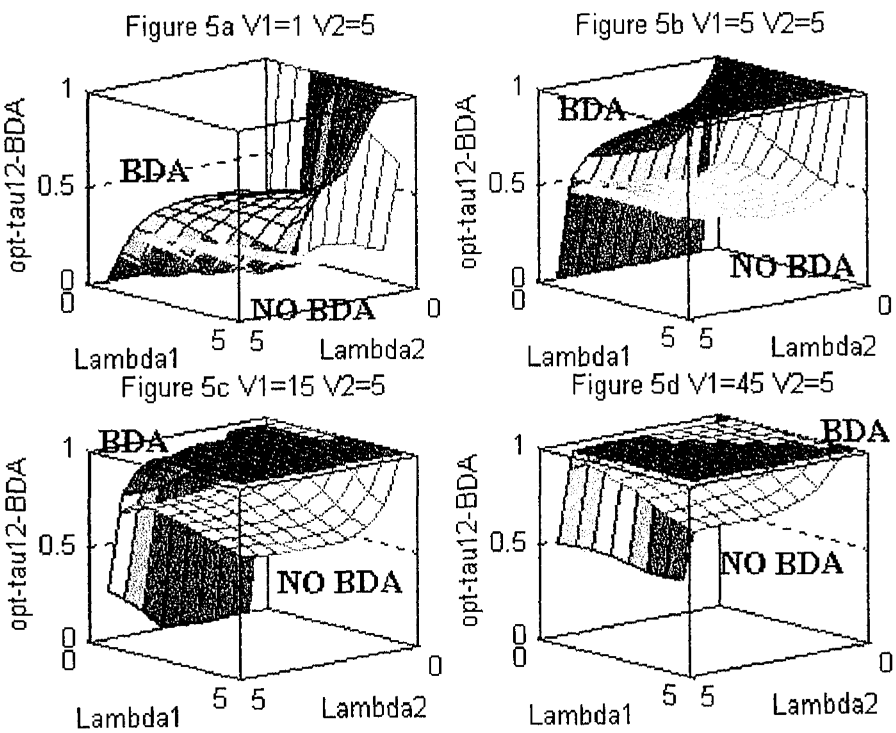

Figure 2-5. The Optimal τ_{12} for BDA System Is Greater (V_1 increases)

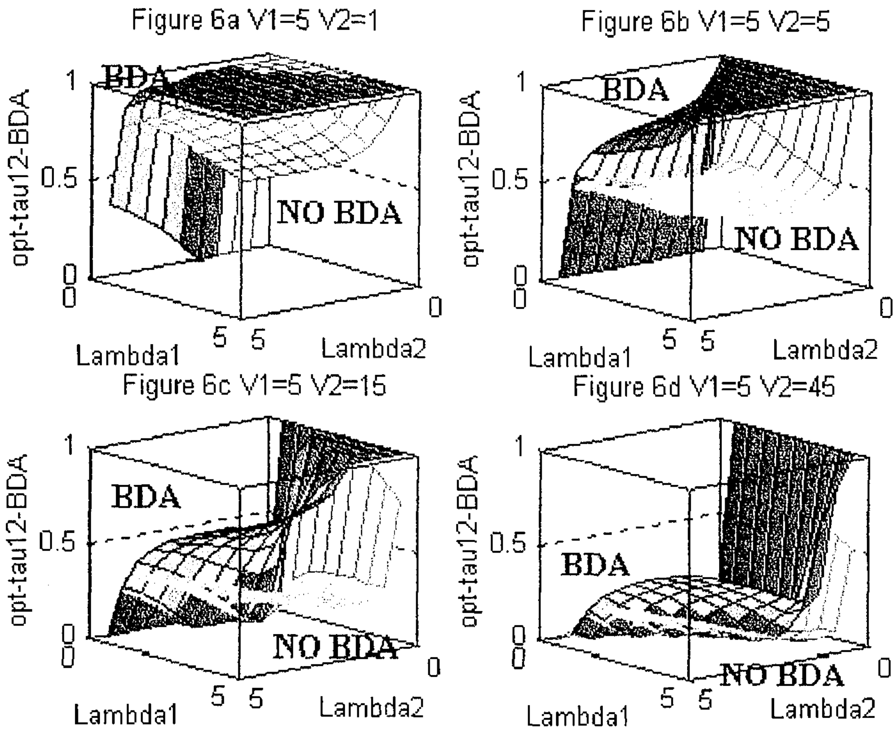

Figure 2-6. The Optimal τ_{12} for BDA System Is Greater (V_2 increases)

D. THE MEASURE OF EFFECTIVENESS (MOE) FOR BDA INFORMATION

Selection of a Measure of Effectiveness (MOE) is perhaps the most important part of any analysis. One of our main objectives is to find a quantitative way to evaluate the value of information. We need a MOE for information that is measurable, quantifiable and measures to what degree the (real) objective is achieved [Ref. 4]. We may choose the time needed to finish a task or the value of the killed targets we achieved. In this paper we select the MOE of information to be the relative fractional improvement in expected return value from the no information case. We define the information gain to be

Gain = (Max (V*p)* - Max (V*n*)) / Max(V*n*) ,

where V*p* is the expected return value with BDA information, V*n* is the expected return value without BDA information, and the maximum is taken with respect to the task switching time τ_{12}.

It is important to determine where BDA information gives us the greatest reward. We also need to find the sensitive parameters. A good MOE should be able to characterize the situation under study. We recognize that the MOE is a function of several variables V_1, V_2, λ_1, λ_2, and the overlap of available kill time windows. We need to know which variable is the most important, and over what ranges the variables should be studied.

The most desirable and direct way to study the relationship between the parameters of the MOE function is to use an analytic formula to find out how the MOE function responds to each factor. Note that the size of an overlapped kill time window is just a relative size with respect to the weapon's kill rate. So we may set a constant overlapped window length to one unit time interval. Thus, we can simplify our problem to four parameters: λ_1, λ_2, V_1 and V_2.

We can use the formula Gain = $(Vp(\tau_p^*) - Vn(\tau_n^*))/ Vn(\tau_n^*)$ to do three-dimensional plots. The optimal expected reward for BDA information $Vp(\tau_p^*)$ is obtained by substituting the optimal threshold τ_p^* into Equation 12. The optimal expected reward for no-BDA information $Vn(\tau_n^*)$ is obtained by substituting the optimal threshold τn^* into Equation 1.

A three-dimensional response surface plot [Ref. 5] can numerically characterize the MOE function with respect to two factors. We choose weapon kill rates λ_1 and λ_2 as the variables so that we can understand how the BDA information gain varies with respect to the weapon efficiency. We may also want to know how the MOE function changes with respect to the change of assigned task value.

The results displayed in Figure 2-7 use the same parameters as before except for the task value $V_1=10$ and $V_2=5$. It shows that the gain from the BDA information is relatively useful for a certain range of parameters. Note that in the region A the weapon efficiency for Task 1 (λ_1) is small (near zero) and the gain from BDA information is also small. Since it is difficult to destroy the first target, the BDA is not useful which is intuitive. In the graph region B, the BDA information gain is higher than in region A. Since λ_1 increases, we have a better chance to get to Task 2. However, in this region both

λ_1 and λ_2 are relatively large and the chance to get both task values is high. Hence, we are indifferent gathering BDA information. Similar reasoning in region C suggests that if the chance to get the value of Task 2 is small, both systems will work on Task 1; thus we gain little from BDA information. In the region D, we can get the best benefit from BDA information which gives us a Gain of about 20 percent.

From the above analysis, we know that if we need to make a decision whether to invest money to improve the weapon efficiency or to gather more information, we have to think carefully whether we can really benefit from our investment.

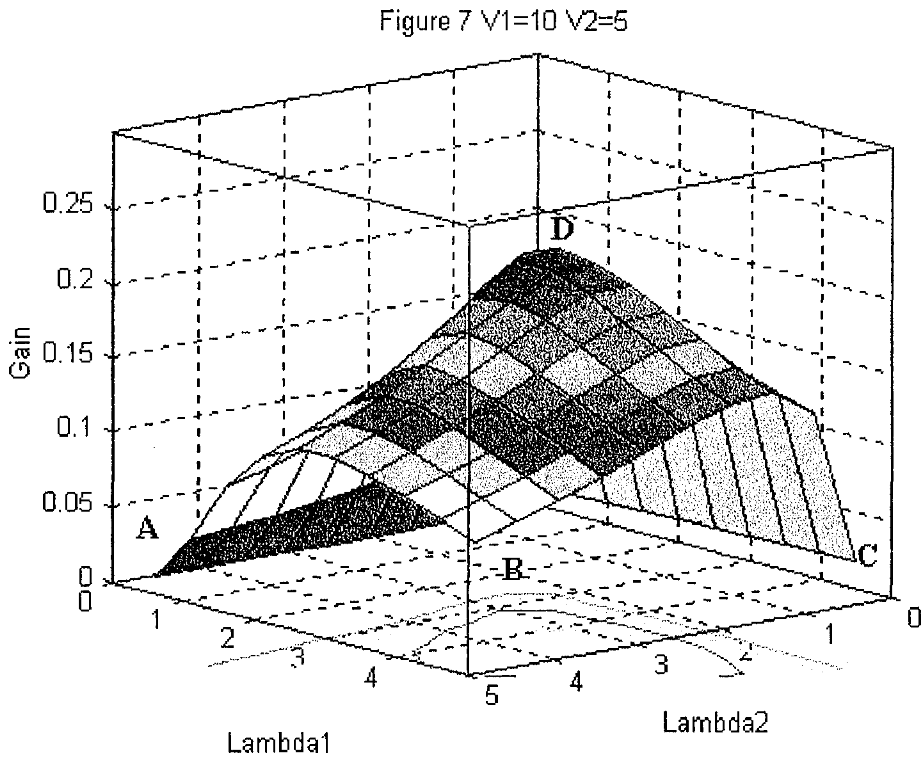

Figure 2-7. BDA Information Gain

The results displayed in Figures 2-8 and 2-9 use exactly the same parameters as Figures 2-5 and 2-6. We notice that in Figure 2-8a the Gain in region A is higher than that the other regions; in this region the value of λ_2 is near zero and V_1 is relatively small. The expected total task value that the systems gets is a small value too. Thus an improvement from BDA information will be significant.

Figure 2-8d displays results for a system that assigns a very high value for Task 1. The information Gain value is below 0.1 all over the whole region. Since both systems allocate most of it's kill time on Task 1, it makes little difference to have BDA information or not to have BDA information.

Figures 2-8 and 2-9 also show that the MOE function is sensitive to the change of parameters. A proper MOE function should be able to reflect the change of the objective function value with respect to the change of parameters. We find that our MOE function is measurable, quantifiable, and measures the degree with which the real objective is achieved.

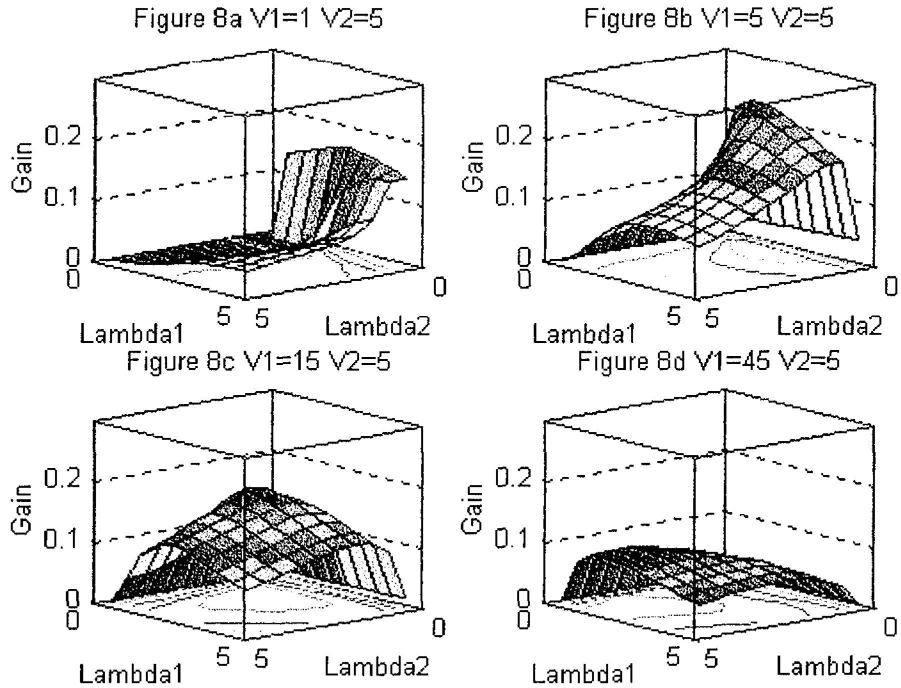

Figure 2-8. BDA Information Gain (V_1 Increases)

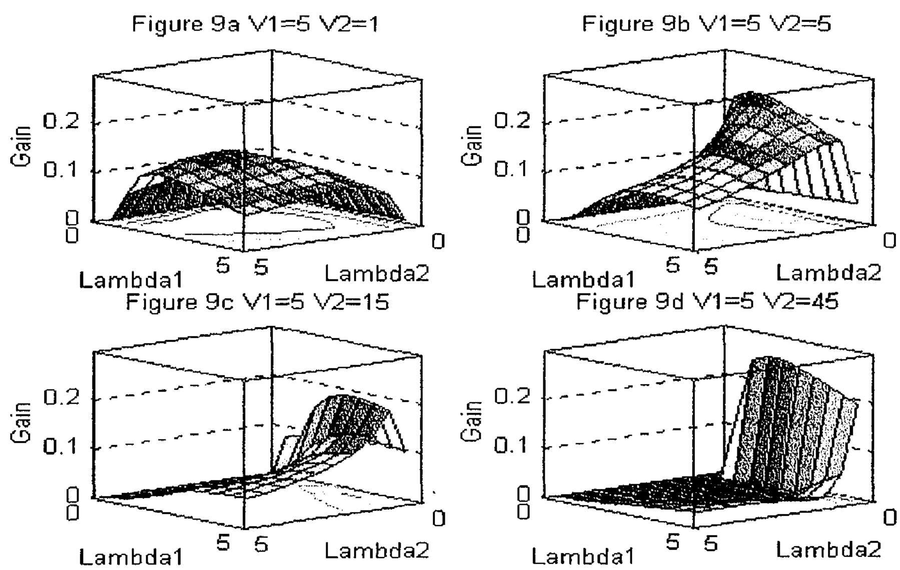

Figure 2-9. BDA Information Gain (V_2 Increases)

III. THE VALUE OF BDA INFORMATION IN ELECTRONIC WARFARE

A. THE BACKGROUND

In military operations, it has become increasingly important to use electronic countermeasures (ECM) and decoys to mislead an enemy's interpretation of radar information. By using electronic jamming techniques the attacking side can imitate radar signals and present false targets and information. Moreover, the enemy can mix real targets with decoys; these when combined with jamming techniques, can greatly increase his probability of success.

In the previous models, we have discovered that if a system pursues an optimal threshold policy to maximize expected reward, it should allocate the precious resource (kill time) to a target that has relatively high value and high kill rate. It is reasonable to assume that the attacker is willing to use ECM and decoys to dupe the defender into thinking a decoy is the more valuable target. If the attacker succeeds, the defender will spend less time on the real target.

The optimal strategy for an attacker is to dupe the defender into working on decoys over the entire overlapped "window." We assume the attacker has two choices: either to launch the decoy first, followed by the real missile; or to launch the missile first, followed by the decoy. Depending on the sequence chosen the battle outcomes will be different. If the attacker chooses to launch the decoy first, he is hoping to dupe the defender into putting off the switching over time for the real missile until the end of the overlapped window. If the attacker chooses to launch the missile first, he is hoping that the defender will choose to switch to the decoy as soon as possible.

We assume that decoy and missile are detected simultaneously, the available kill time window for both targets are completely overlapped. Thus, the overlapped window is defined as $w = t_{12} = t_{22}$ (See Figure 2-1). We also assume that the defender is unaware of being duped. Proceeding with the following analysis as in the previous models, if the decoy is launched first, we assume that the decoy is the first target presented to the defender. If the decoy is launched second, then the second target presented to the defender is the decoy. By BDA information is meant the information that the first target has been killed when it is killed.

B. MODELS FOR DEFENDER UNAWARE OF BEING DUPED

1. Case I : The Attacker's Decoy is First, Followed by the Missile

The decoy and missile have fixed rates of being killed of λ_d and λ_m respectively. The missile has a fixed value of V_m . Since the defender has been duped, he will be duped into assigning a duped value $V_d^{'}$ to the decoy and choosing an optimal threshold time τ_{dm} to switch tasks. Because the actual reward value of the decoy is zero, the more value that the defender assigns to the decoy the more successful will be the attacker's ECM operation.

By varying the decoy's duped value $V_d^{'}$, we can compare systems with and without BDA information to see how they work under different levels of ECM operation. The *actual expected reward value* of a system without BDA information can be obtained by rewriting Equation 1 with $V_d = 0$, $t_{22} = w$. The actual expected value the defender gets is:

$$V_n\left(\tau_{dm}\right) = V_m\left[1 - e^{-\lambda_m\left(w - \tau_{dm}\right)}\right] \quad \mathbf{(31)}$$

where τ_{dm} is the duped optimal threshold for the no-BDA information system.

If the defender has BDA information, from Equation 12 setting $V_d = 0$, $t_{12} = t_{22} = w$, and $t_{21} = t_{11} = 0$, the actual expected value the defender gets is:

$$V_p(\tau_{dm}) = V_m\left[\left(1 - e^{-\lambda_m(w-\tau_{dm})}\right)e^{-\lambda_d \tau_{dm}} + \left(1 - e^{-\lambda_d \tau_{dm}}\right) - e^{-\lambda_m w}\frac{\lambda_d}{\lambda_d - \lambda_m}\left[1 - e^{-\tau_{dm}(\lambda_d - \lambda_m)}\right]\right] \quad \mathbf{(32)}$$

where τ_{dm} is the duped threshold for the BDA information system.

Figure 3-1 displays results for $\lambda_d = 2$, $\lambda_m = 1$,$V_m = 5$, $w = 1$.

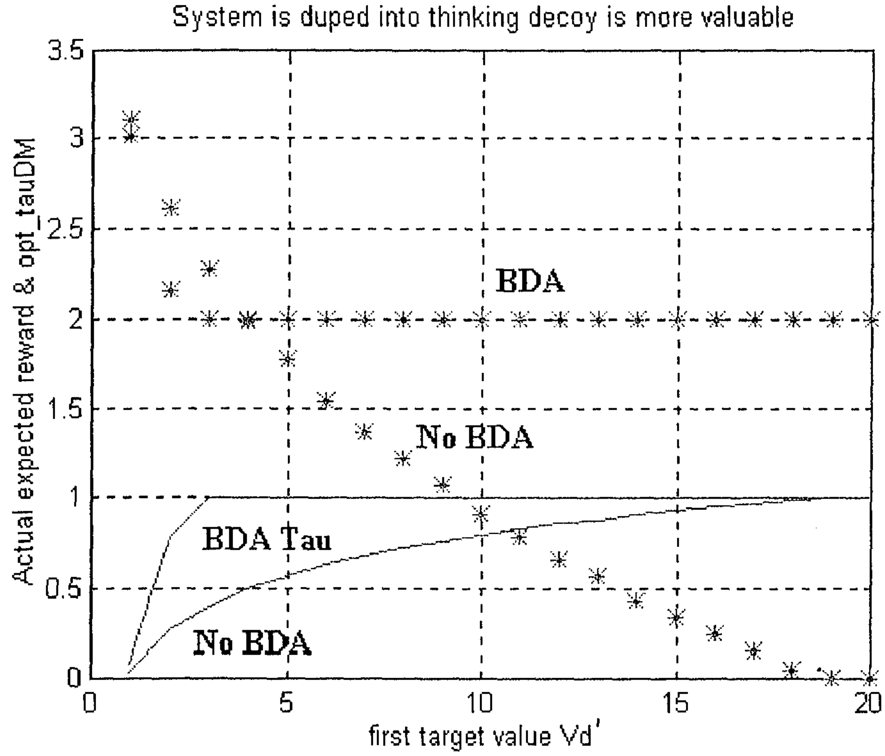

Figure 3-1 System Duped to Think Decoy is More Valuable

The x axis is the decoy's duped value which is assigned by the defender. We vary the duped value $V_d^{'}$ from 1 to 20 to represent different levels that the defender has been duped. The y axis is a combined scale which includes the duped optimal τ_{dm} (from 0 to 1) and the actual expected reward value with both BDA and no-BDA information system.

In Figure 3-1, we find that when $V_d^{'}$ is less than 4, the actual expected return value of a system with BDA information is less than that a system without BDA information.

There are two reasons why BDA information may not be advantageous: First, from the previous model we know that a system with BDA information tends to make its changeover threshold higher than a system without BDA information. This property causes a system which has BDA information to waste more time on the decoy. Second, the decoy is not likely to be destroyed within a short time interval, so the BDA information will not be helpful.

The no-BDA information system has shortcomings when the system is duped into thinking that a decoy is more important than a missile. A system with no-BDA information, may set its threshold towards the end of the window, resulting in the no-BDA system returning no reward value since the system has spent all its time on a decoy.

For a system with BDA information, even though it is duped into thinking a decoy is a missile it can still receive some reward value V_m from the real target since it uses min (T_d, τ^*_{dm}) strategy. If the decoy is destroyed, the system will immediately start to work on the real target. The worst case for the system with BDA information is to set its threshold to the end of window ($\tau_{dm} = w$). In this situation, the expected return value is obtained by conditioning on T_d.

$$
\begin{aligned}
V_p &= V_m \int_0^w \left(1 - e^{-\lambda_m (w-x)}\right) e^{-\lambda_d x} \lambda_d dx \\
&= V_m \left[(1 - e^{-\lambda_d w}) - e^{-\lambda_m w} \frac{\lambda_d}{\lambda_d - \lambda_m} (1 - e^{-(\lambda_d - \lambda_m) w}) \right]
\end{aligned}
\qquad \textbf{(33)}
$$

From Equation 33 we know that the larger λ_d is, the easier it is to kill the decoy and the more expected return value a system with BDA information will receive. Figure 3-2 displays results for the same parameters as in Figure 3-1 except that the kill rate for the decoy is $\lambda_d = 4$. It shows the actual expected reward (V_p=2.6) is higher than that of previous example (V_p=2).

From the above analysis the models give some insights into the value of BDA information in an ECM environment. The attacker should make the defender think that the decoy is very valuable and easily destroyed. The attacker should create a decoy that take a maximum time to destroy. For the defender who has BDA information, the information should improve his efficiency in destroying the decoy. Thus, the defender can benefit from BDA information.

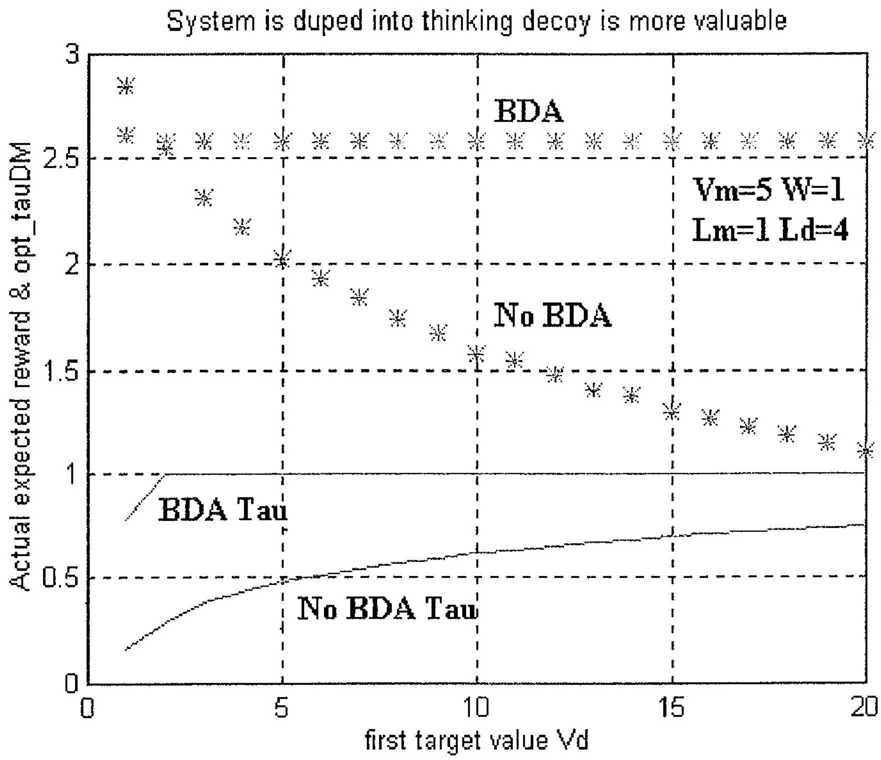

Figure 3-2 System Duped to Think Decoy is More Valuable $\lambda_d = 4$

2. Case II : The Attacker's Missile Is First, Followed By The Decoy

The attacker launches the real missile first then launches the decoy. The missile and decoy have fixed rates of being killed of $\lambda_m = 1$ and $\lambda_d = 2$ respectively. The missile has a fixed value $V_m = 5$. If the defender is duped into thinking that the second target decoy is a missile, the system with BDA information will have a higher actual return value. Both models, with and without BDA information use the formula for "actual reward value " of $V_m\left(1-e^{-\lambda_m \tau_{md}}\right)$. However, the duped optimal τ_{md} value for a BDA information is higher than that of a no-BDA information system. So the actual expected reward for a system with BDA information is higher than that for a system without BDA information. If both systems are duped into thinking that a decoy is high value target, then both will receive no value, since they both will set the duped optimal threshold τ_{md} to zero. Figure 3- shows both the BDA and the no-BDA system receive zero value when duped into thinking a decoy is the most important target.

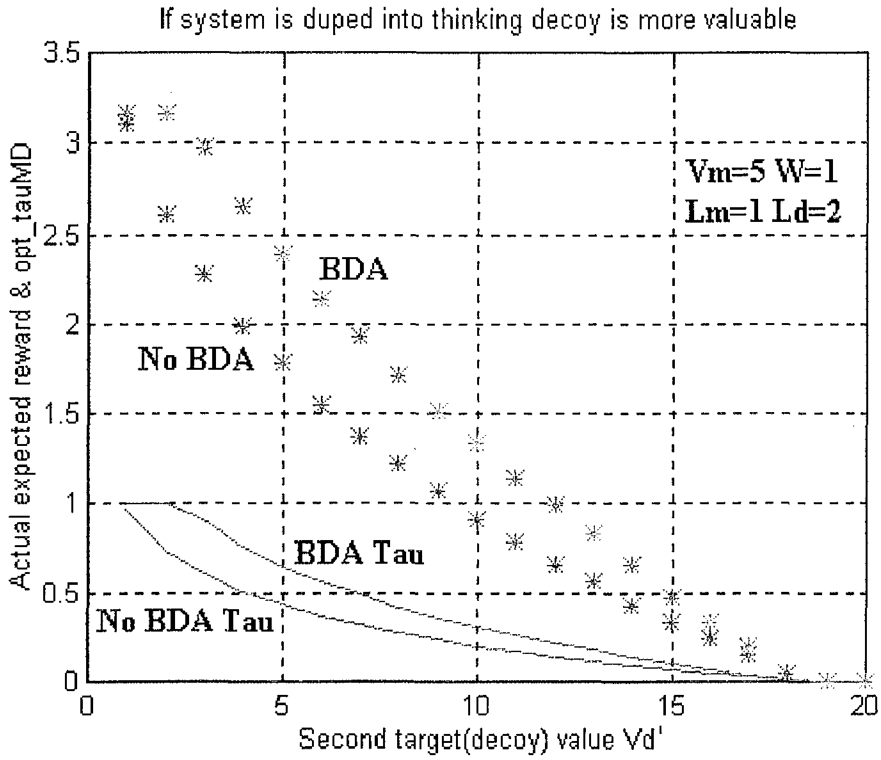

Figure 3-3. BDA and No-BDA System Receive Zero Value

3. Case III Countermeasure by Maxmin Expected Reward Criteria

Assume that the defender does not have enough information to correctly differentiate between a decoy and a missile. However, the defender knows that the attacker may take advantage by using a decoy. The defender must make a decision to choose an optimal threshold τ which will not be affected greatly by the enemy's ECM efforts. We assume that a conservative defender will use a "maxmin criteria." This means the defender will choose a threshold among all possible thresholds with the "best" of the worst outcome values.

Assume there is only one missile and one decoy and that they appear simultaneously. Assume their available kill time windows are completely overlapped. First, consider a system without BDA information. It will choose a threshold τ to engage the first target. It will receive a reward value by destroying the real target. If the system engages the missile first, the kill time allocated to the missile will be τ. The expected reward value in this case will be:

$$V_{n1}(\tau) = V_m\left[1 - e^{-\lambda_m \tau}\right] \tag{34}$$

If the system engages the decoy first, then the kill time allocated to the missile will be the remaining time $(w-\tau)$. The expected reward value will be:

$$V_{n2}(\tau) = V_m\left[1 - e^{-\lambda_m (w-\tau)}\right] \tag{35}$$

If the no-BDA information defender uses a "maxmin criteria," then his optimal expected reward value will be $V_n(\tau^*) = \mathrm{Max}\,(\,\mathrm{Min}\,(V_{n1}(\tau)),\ \mathrm{Min}\,(V_{n2}(\tau))\,)$, where the minimum is taken over all switching times.

Following similar reasoning yields for a system with BDA information an actual reward value of:

$$V_{b1}(\tau) = V_m\left[1 - e^{-\lambda_m \tau}\right] \tag{36}$$

or

$$V_{b2}(\tau) = V_m\left[\left(1 - e^{-\lambda_m (w-\tau)}\right)e^{-\lambda_d \tau} + \left(1 - e^{-\lambda_d \tau}\right) - e^{-\lambda_m w}\frac{\lambda_d}{\lambda_d - \lambda_m}\left[1 - e^{-\tau(\lambda_d - \lambda_m)}\right]\right] \tag{37}$$

The optimal expected reward value for BDA system will be:

$$V_b(\tau^*) = \mathrm{Max}\,(\,\mathrm{Min}\,(V_{b1}(\tau)),\ \mathrm{Min}\,(V_{b2}(\tau))\,). \tag{38}$$

In Figure 3-4 we use the same parameters λ_m = L2 =1 ,V_m = 5, V_d = 0, w = 1. In Figures 3-4b to 3-4d situations the BDA information system has a decoy kill rate (λ_d = L1) equal to 5, 3 and 0.2 unit respectively. The Figures display results that a system with BDA information using the maxmin criteria will tend to set its threshold greater than half of the available kill window provided the kill rate of a decoy is high. In this manner the BDA system can also get more expected value than a system without BDA information. From Equation 37 if the λ_d is close to zero then the equation is approximately equal to Equation

35 . This means when the BDA information system uses the maxmin criteria it is at least as good as the no-BDA information system(Figure 3-4a).

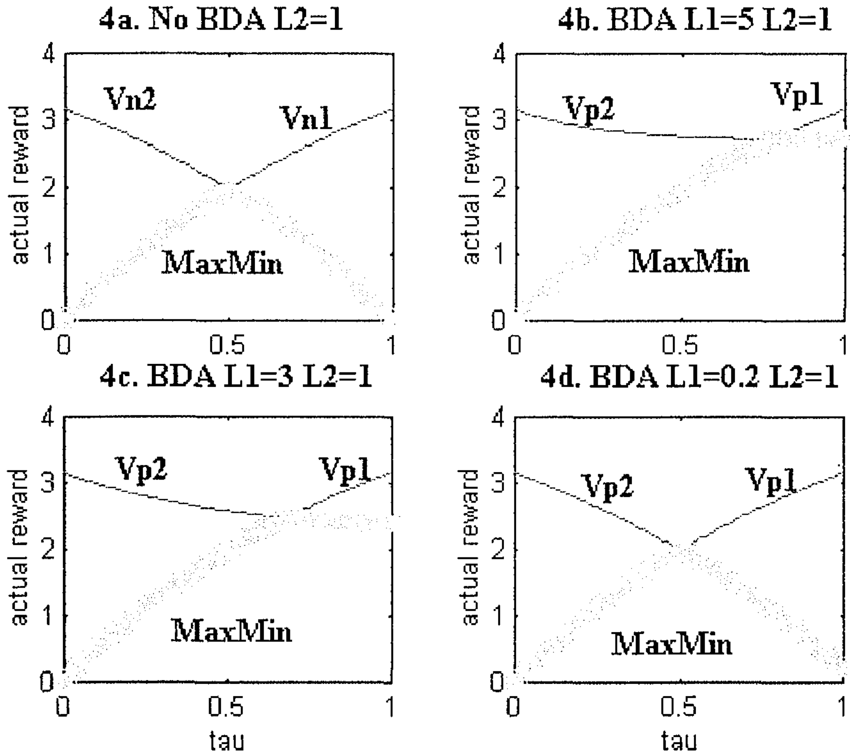

Figure 3-4 System With BDA Information Using Maxmin Criteria

4. Case IV Countermeasure by Hard Kill

The proceeding discussion of enemy's electronic countermeasure invokes one of the most important problems of operations research. It is possible for the defender to devote time to gathering more identification information and to use that information to ensure a "hard kill," i.e. to use his weapon to shoot down any available targets. If electronic support measures (ESM) for identifying targets are not available, the decision maker must make a decision as whether to shoot both targets or only a single target.

It is important to provide a sensitivity analysis of different weapon efficiencies for the decoy target. The purpose of the analysis is to provide the decision maker with a basis for decision. Thus the decision maker can make his decision based on his own weapon efficiency.

Assume that the defender believes his weapon can easily kill the enemy's decoy. The defender wishes to use his weapon to destroy any targets and reduce the possibility that a missed target could deliver its warhead. Because identification information is not available, the defender has to subjectively assign values to targets. If the defender believes that the target is a decoy he will assign a low task value to that target. Let's assume that the value or utility of destroying this decoy is 0.5. If the defender believes that the target is a missile, he will assign a task value of 5 units. For the discussion below, assume there are two incoming targets, one is the real missile and the other a decoy.

The enemy's tactical alternative will be defined as a random event X that has states of θ_1 and θ_2. Use θ_1 to represent the event that the enemy launches the decoy first then the missile and θ_2 to represent the attacker's opposite launching sequence.

The defender's tactical decision for both BDA and no-BDA systems will be to increase or decrease the threshold time to switch to Task 2. Since the no-BDA system will pursue the optimal threshold policy, if the defender believes that the decoy comes first, by assigning a low value to the first target the system will decrease the threshold time automatically. We denote the decreased optimal time threshold to be τ_{dm}*; this policy will be denoted by d_1. If the defender believes that the missile comes first, by assigning a high value to the first target the system will decrease the threshold time automatically. We denote the decreased optimal threshold time to be τ_{md}*; this policy will be denoted by d_2. A similar threshold behavior can be applied to the BDA system. However, the decreased threshold time decision (d_1) for BDA information system will be determined by the min(T_d , τ_{dm}*), this policy increased threshold time decision (d_2) will be determined by the min(T_m ,τ_{md}*) .

The result of the defender's decision $d_i \in D$ and attacker's actions $\theta_j \in X$ will generate the payoff of R_{ij}. The payoff matrices for no-BDA and BDA information systems are shown in Table 1 and Table 2 respectively. The payoff for both systems can be calculated by using the equations appearing in Appendix A.

Figure 3-5 displays results for the attacker launch sequence: the decoy comes first followed by the missile and the parameters $\lambda_m = 1$, $V_m = 5$, $V_d = 0.5$, $t_{12} = t_{22} = 1$, $t_{11} = t_{21} = 0$. We vary the decoy kill rate λ_d from 0.1 to 15. When the kill rate for the decoy λ_d is above 5 units, the actual reward value(R_{21BDA}) for the BDA information system which misidentified the target sequence is nearly the same as the actual reward value(R_{11BDA}) if the BDA system correctly identified the target. Since the defender believes that the missile comes first and reserves more of the time for first target (decoy) the BDA information system with a high decoy kill rate can ameliorate the effect of the misidentification. However, the best case for no-BDA information can only achieve 1 unit actual expected reward if it misidentifies the attack sequence.

Attacker's Action→ Defender's Action↓	Use θ_1 (Decoy / Missile) Sequence to Attack	Use θ_2 (Missile / Decoy) Sequence to Attack
Use d_1 (Decoy / Missile) Sequence to Defend	Correctly Apply Threshold τ_{dm}^* : R_{11}	Incorrectly Apply Threshold τ_{dm}^* : R_{12}
Use d_2 (Missile / Decoy) Sequence to Defend	Incorrectly Apply Threshold τ_{md}^* : R_{21}	Correctly Apply Threshold τ_{md}^* : R_{22}

Table 1. Payoff Matrix for No-BDA System

Attacker's Action→ Defender's Action↓	Use θ_1 (Decoy / Missile) Sequence to Attack	Use θ_2 (Missile / Decoy) Sequence to Attack
Use d_1 (Decoy / Missile) Sequence to Defend	Correctly Apply Threshold $\mathrm{Min}(T_d, \tau_{dm}^*)$: R_{11BDA}	Incorrectly Apply Threshold $\mathrm{Min}(T_m, \tau_{dm}^*)$: R_{12BDA}
Use d_2 (Missile / Decoy) Sequence to Defend	Incorrectly Apply Threshold $\mathrm{Min}(T_d, \tau_{md}^*)$: R_{21BDA}	Correctly Apply Threshold Min $\mathrm{Min}(T_m, \tau_{md}^*)$: R_{22BDA}

Table 2. Payoff Matrix for BDA System

Figure 3-6 displays results for the attacker launch sequence: the missile comes first followed by the decoy. The parameters are the same as those of Figure 3-5. We find that if the kill rate for the decoy λ_d is above 10 units, the actual reward value

(R_{12BDA}) for the BDA information system which has misidentified the target sequence is about the same as the actual reward value (R_{11BDA}) if the system correctly identified the sequence. Since the defender believes that the decoy comes first and allocates less time for first target (decoy) the BDA information system need a higher efficiency of $\lambda_d = 10$ rather than $\lambda_d = 5$ to make up for the identification mistake. However, the best case for no-BDA information can only achieve 1 unit actual expected reward if it misidentifies the attack sequence.

From the above analysis, we conclude that if a system has BDA information and a high weapon efficiency, it is possible for the system to counter the decoy by killing both targets. However, for a system with no-BDA information the chance to successfully counter the decoy effect is less than a system with BDA information.

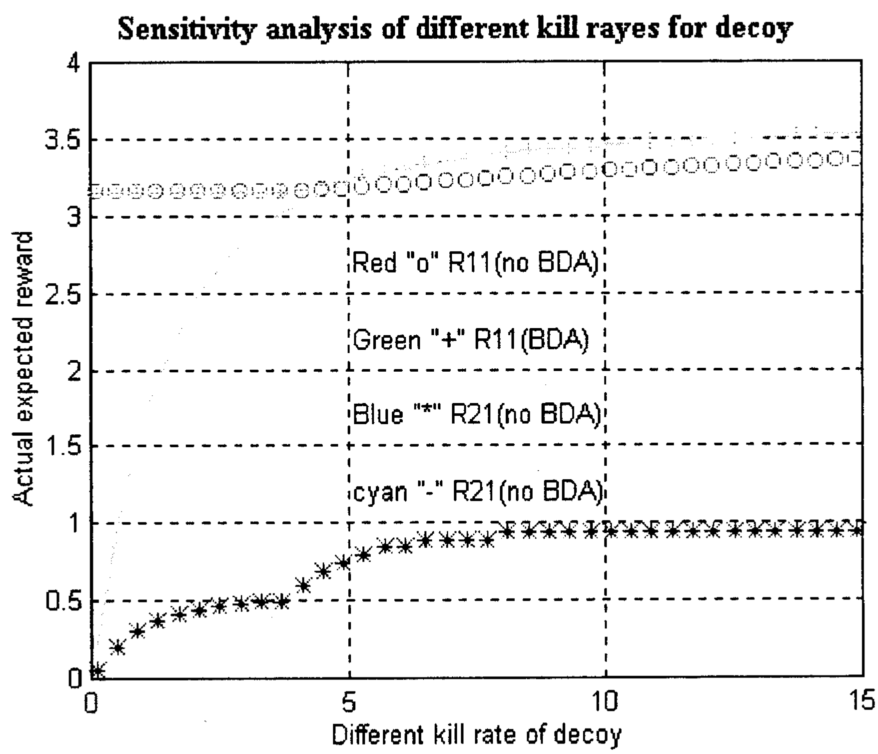

Figure 3-5 Weapon Efficiency Sensitivity Analyses for Situation in Which Decoy Comes First

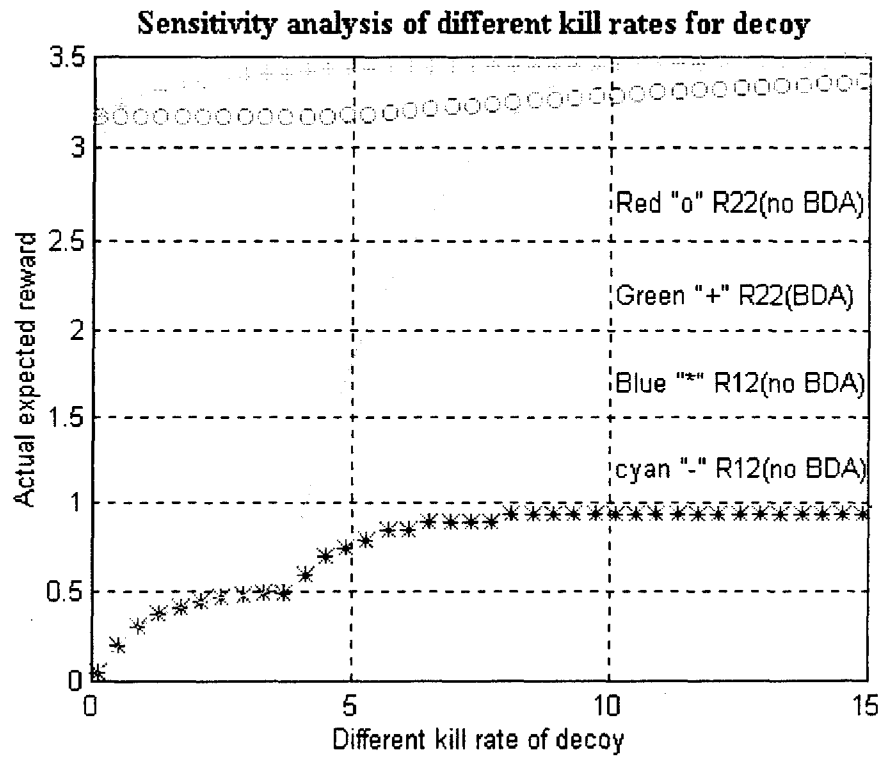

Figure 3-6. Weapon Efficiency Sensitivity Analyses for Situation in Which Missile Comes First

IV. CONCLUSIONS AND POSSIBLE FOLLOW-ON RESARCH

A. CONCLUSIONS

Recently the People's Republic of China has been using their M-9 medium-range, mobile-launched ballistic missiles to intimidate my country, Taiwan, the Republic of China. The M-9 ballistic missile which can carry a 1,100 lb. single warhead has been launched and splashed barely 20 nautical miles away from Taiwan's main seaports, Keelung and Kaohsiung. It is reported that bank depositors have withdrawn $370 million from the Taiwan banks. The country's economic outlook has been greatly affected by the ballistic missile attacks. Moreover, the political confrontation is not just between Taiwan and mainland China but also involves the United States.

Again this is a demonstration that the threat from theater ballistic missiles is growing. Such weapons can play a major political role in a regional conflict. The reasons for a theater missile defense project are: (1) to protect allies and troops deployed overseas in the theater conflicts; (2) to discourage global ballistic missile proliferation; (3) to reduce the chance that an ally is politically affected by the threat of a missile attack. The United States should devote more efforts to theater ballistic missile defense.

However, theater missile defense requires major investment. The proper evaluation of weapon and information efficiency should be carefully studied. This thesis investigated the effects and value of battle damage assessment information availability in the defense against sequential missile attacks. It was found that both no-BDA and BDA information systems may both allocate their scarce resources, i.e. the available kill time, to achieve the best battle outcome, but that BDA can provide an advantage to the defender.

For the purpose of investigating, and to quantify the value of information, we define a measure of effectiveness(MOE) for information. Without a proper MOE function to provide quantitative insights into feasibility and critical physical factors, a proper decision under uncertainty cannot be made. Developing a MOE function and applying it to the question of allocation of scarce resources in the light of the available information is not only for the purpose of information evaluation, it also serves the purpose of assisting in the selection of a sound tactical strategy for decision making. The MOE function can provide insight into how the various system parameters interact and how sensitive they are to changes. The MOE function demonstrates that the effect of information is measurable, and quantifiable, and that a MOE can quantify the degree that the real objective is achieved. It can also help in understanding the trade-offs between weapon efficiency and the value information. In conclusion, the MOE function can be used for BDA information evaluation.

In Chapter III, we compare the value of BDA information and no-BDA information in certain situations involving electronic warfare (EW). Under different levels of deception, the BDA and no-BDA information will result in different rewards. A careful design of a system must be made to really benefit from BDA information. In the design it should be kept in mind that BDA is not a panacea. Depending on the enemy's strategies the outcomes will be different with BDA or with no-BDA system. However, it is always desirable to increase the first target kill rate to benefit from the BDA information.

B. POSSIBLE FOLLOW-ON RESEARCH

In modern warfare, information plays an important role. In this thesis, the discussion is primarily about the value of BDA information. However, in an EW environment target identification information can play a key role. Bayesian decision analysis could be applied to assist in the decision as to whether to develop assets to acquire or to improve weapon efficiency. If there is reliable target identification information, a defender may be able to choose to only shoot the real target with high probability and thus to save limited kill time and ammunitions. This issue would involve two kinds of conditional probability distribution. First, is the decision probability of particular outcome x given a particular forecast f. Second, is the likelihood probability of particular forecast f given a particular outcome x [Ref. 6]. The expected reward from using identification information to shoot at only one target might be at least as good as using a maxmin criteria, provided the likelihood probability specification is accurate and the data on the possible targets are adequate. However, the defender can also change his threshold to maximize the expected reward. An appropriate model may be formulated as a two-person zero sum game.

In this thesis, it is assumed that the target acquisition time is zero. However, in real life, it is possible to spend significant time on such a task. A problem will be how to best allocate the acquisition and kill time. How does a BDA and no-BDA information system respond?

Finally, suppose there are M incoming missiles of N types with each having a different task value. The available time to work on tasks is limited and the available shooters (servers) are also limited. The question will be how to maximize the expected reward subject to the limited resources.

APPENDIX

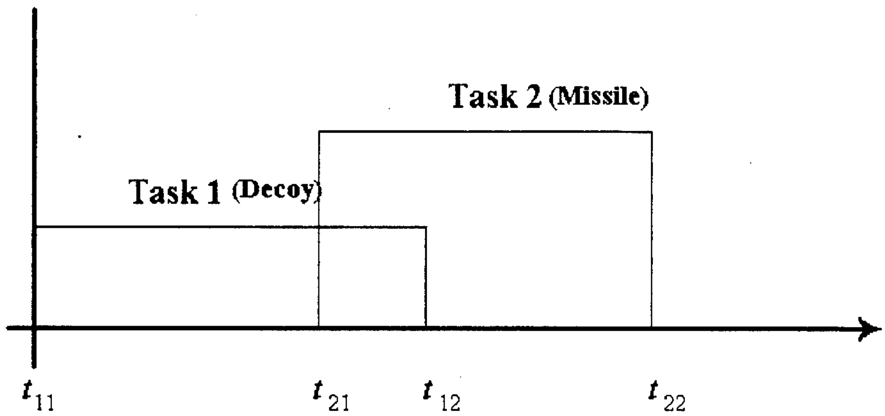

Figure 1. Task 1 come first ,Task 2 come after.

A. NO-BDA INFORMATION CASE

In this section, it is assumed that the decoys is launched first and the missile second. Assume that the decoy and missile have value V_d and V_m and kill rates λ_d and λ_m respectively. If the system can correctly identify the sequence the expected return R_{11} is.

$$V(\tau_{dm}) = V_d + V_m - \left[V_d e^{-\lambda_d \tau_{dm}} + V_m e^{-\lambda_m (t_{22} - \tau_{dm})}\right] \quad \textbf{(A1)}$$

By using τ_{dm}^* the optimal payoff will be :

$$R_{11} = V(\tau_{dm}^*) = V_d + V_m - \left[V_d e^{-\lambda_d \tau_{dm}^*} + V_m e^{-\lambda_m (t_{22} - \tau_{dm}^*)}\right] \quad \textbf{(A2)}$$

If the defender is duped into thinking that the missile comes first then the decoy comes after, the system will switch tasks after τ_{md}* and the expected return will be:

$$R_{21} = V(\tau_{md}^*) = V_d + V_m - \left[V_d e^{-\lambda_d \tau_{md}^*} + V_m e^{-\lambda_m (t_{22} - \tau_{dm}^*)}\right] \quad \textbf{(A3)}$$

B. WITH BDA INFORMATION AFTER MIN(T_d, τ_{dm})

In this section it is assumed the decoy is launched first and the missile second. The policy is to switch to Task 2 (missile) after min(T_d ,τ_{dm}). The expected reward value can be obtained by conditioning on $T_d = t_d$

$$
\begin{aligned}
V(\tau_{dm};t_d) &= V_d + V_m\left(1-e^{-\lambda_m(t_{22}-t_{21})}\right) && t_d \le t_{21} \\
&= V_d + V_m\left(1-e^{-\lambda_m(t_{22}-t_d)}\right) && t_{21} < t_d < \tau_{dm} \le t_{12} \\
&= V_m\left(1-e^{-\lambda_m(t_{22}-\tau_{dm})}\right) && t_{12} \le \tau_{dm} < t_d
\end{aligned}
\tag{A4}
$$

Removing the condition, the expected payoff R_{11} will be:

$$
\begin{aligned}
R_{11BDA} = V(\tau_{dm}^*) = V_d(1-e^{-\lambda_d\tau_{dm}^*}) + V_m\Big[& [(1-e^{-\lambda_d t_{21}})(1-e^{-\lambda_m(t_{22}-t_{21})}) \\
& + (1-e^{-\lambda_m(t_{22}-\tau_{dm}^*)})e^{-\lambda_d\tau_{dm}^*} + (e^{-\lambda_d t_{21}} - e^{-\lambda_d\tau_{dm}^*}) \\
& - e^{-\lambda_d t_{21}} e^{-\lambda_m(t_{22}-t_{21})} \frac{\lambda_d}{\lambda_d-\lambda_m}[1-e^{-(\lambda_d-\lambda_m)(\tau_{dm}^*-t_{21})}]\Big]
\end{aligned}
\tag{A5}
$$

If the defender is duped into thinking Task 1(decoy) is Task 2(missile) then he would switch task after min(T_d, τ_{md}^*). We can condition on T_d then the expected payoff R_{21} would be:

$$
\begin{aligned}
V(\tau_{md}^*;t_d) &= V_d + V_m(1-e^{-\lambda_m(t_{22}-t_{21})}) && t_d \le t_{21} \\
&= V_d + V_m(1-e^{-\lambda_m(t_{22}-t_d)}) && t_{21} < t_d \le \tau_{md}^* \le t_{12} \\
&= V_m(1-e^{-\lambda_m(t_{22}-\tau_{md}^*)}) && t_{12} \le \tau_{md}^* < t_d
\end{aligned}
\tag{A6}
$$

Removing the condition:

$$
\begin{aligned}
R_{21BDA} = V(\tau_{md}^*) = V_d(1-e^{-\lambda_d\tau_{md}^*}) + V_m\Big[& (1-e^{-\lambda_d t_{21}})(1-e^{-\lambda_m(t_{22}-t_{21})}) \\
& + (1-e^{-\lambda_m(t_{22}-\tau_{md}^*)})e^{-\lambda_d\tau_{md}^*} + (e^{-\lambda_d t_{21}} - e^{-\lambda_d\tau_{md}^*}) \\
& - e^{-\lambda_d t_{21}} e^{-\lambda_m(t_{22}-t_{21})} \frac{\lambda_d}{\lambda_d-\lambda_m}[1-e^{-(\lambda_d-\lambda_m)(\tau_{md}^*-t_{21})}]\Big]
\end{aligned}
\tag{A7}
$$

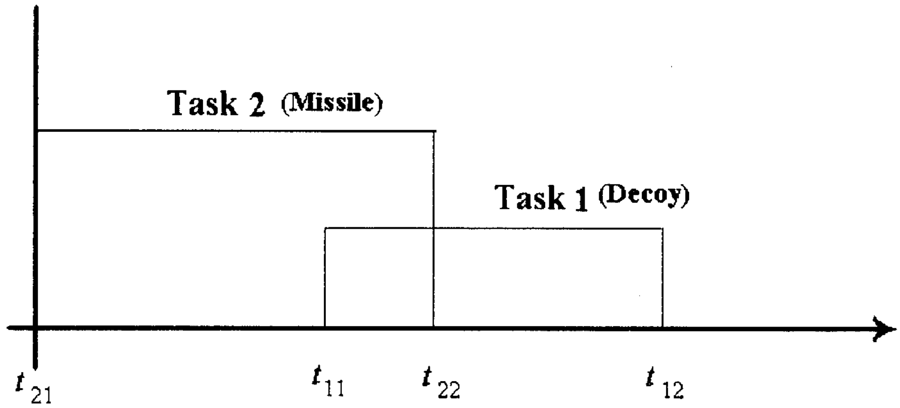

Figure 2. Task 2 comes first;Task1 comes after.

C. NO-BDA INFORMATION CASE

In this section, it is assumed that the missile is launched first and then the decoy. Assume that the decoy and missile have values V_d and V_m and kill rates λ_d and λ_m respectively. If the defender correctly identifies the sequence the expected return R_{22} is

$$\begin{aligned} V(\tau_{md}) &= V_m\left(1-e^{-\lambda_m\tau_{md}}\right)+V_d\left(1-e^{-\lambda_d(t_{12}-\tau_{dm})}\right) \\ &= V_d+V_m-\left[V_m e^{-\lambda_m\tau_{md}}+V_d e^{-\lambda_d(t_{12}-\tau_{md})}\right] \end{aligned} \qquad \textbf{(A8)}$$

$$R_{22}=V\left(\tau_{md}^*\right)=V_d+V_m-\left[V_m e^{-\lambda_m\tau_{md}^*}+V_d e^{-\lambda_d(t_{12}-\tau_{md}^*)}\right] \qquad \textbf{(A9)}$$

If the defender is duped into thinking Task 2(decoy) is Task 1(missile) then he will use τ_{dm}^*, in this case:

$$R_{12}=V\left(\tau_{dm}^*\right)=V_d+V_m-\left[V_m e^{\lambda_m\tau_{dm}^*}+V_d e^{-\lambda_d\left(t_{12}-\tau_{dm}^*\right)}\right] \qquad \textbf{(A10)}$$

D. WITH BDA INFORMATION AFTER MIN(T_m, τ_{md})

In this section, it is assumed that the missile is launched first and then the decoy. Suppose that the defender switch to Task 2 after min(T_m , τ_{md}). The expected reward value can be obtained by conditioning on $T_m = t_m$:

$$\begin{aligned}
V(\tau_{md};t_m) &= V_m + V_d\left(1-e^{-\lambda_d(t_{12}-t_{11})}\right) && t_m \le t_{11} \\
&= V_m + V_d\left(1-e^{-\lambda_d(t_{12}-t_d)}\right) && t_{11} < t_m < \tau_{md} \le t_{22} \\
&= V_d\left(1-e^{-\lambda_d(t_{12}-\tau_{md})}\right) && t_{11} \le \tau_{md} < t_m
\end{aligned} \tag{A11}$$

Removing the condition then the expected payoff R_{22} will be:

$$\begin{aligned}
R_{22BDA} &= V(\tau_{md}^*) = V_m\left(1-e^{-\lambda_m\tau_{md}^*}\right) + V_d\Big[\left(1-e^{-\lambda_m t_{11}}\right)\left(1-e^{-\lambda_d(t_{12}-t_{11})}\right) \\
&+ \left(1-e^{-\lambda_d(t_{22}-\tau_{md})}\right)e^{-\lambda_m\tau_{md}^*} + \left(e^{-\lambda_m t_{11}}e^{-\lambda_m \tau_{md}^*}\right) \\
&- e^{-\lambda_m t_{11}}\, e^{-\lambda_d(t_{12}-t_{11})}\frac{\lambda_m}{\lambda_m-\lambda_d}\left[1-e^{-(\lambda_m-\lambda_d)(\tau_{md}^*-t_{11})}\right]\Big]
\end{aligned} \tag{A12}$$

If the defender is duped into thinking Task1 (missile) is Task2 (decoy) the defender would use $\min\left(T_m, \tau_{dm}^*\right)$ in this case:

$$\begin{aligned}
V(\tau_{dm}^*;t_m) &= V_m + V_d(1-e^{-\lambda d(t_{12}-t_{11})}) && t_m \le t_{11} \\
&= V_m + V_d(1-e^{-\lambda d(t_{12}-t_m)}) && t_{11} < t_m \le \tau_{dm}^* \le t_{22} \\
&= V_d(1-e^{-\lambda_d(t_{12}-\tau_{dm}^*)}) && t_{11} \le \tau_{dm}^* < t_m
\end{aligned} \tag{A13}$$

Removing the condition the expected payoff R_{12} will be:

$$\begin{aligned}
R_{12BDA} &= V(\tau_{dm}^*) = V_m(1-e^{-\lambda_m \tau_{dm}^*}) + V_m\Big[(1-e^{-\lambda_m t_{11}})(1-e^{-\lambda_d(t_{12}-t_{11})}) \\
&+ (1-e^{-\lambda_d(t_{12}-\tau_{dm}^*)})e^{-\lambda_m \tau_{dm}^*} + (e^{-\lambda_m t_{11}} - e^{-\lambda_m \tau_{dm}^*}) \\
&- e^{-\lambda_m t_{11}}e^{-\lambda_d(t_{12}-t_{11})}\frac{\lambda_m}{\lambda_m-\lambda_d}[1-e^{-(\lambda_m-\lambda_d)(\tau_{dm}^*-t_{11})}]\Big]
\end{aligned} \tag{A14}$$

LIST OF REFERENCES

1. Nicholas, Ted. and Rossi, Rita. , U. S. Missile Data Book, 18th ed. 1994.

2. Weapon Systems, United States Army, p. 57, 1995.

3. Ross, S.M. , Introduction to Probability Models, 5th ed. p. 91-100, Academic Press, Inc. , 1993.

4. Naval Operations Analysis, 2th ed. p. 13, United States Naval Institute, 1989.

5. Law, Averill M. and Kelton, W. David. , Simulation Modeling and Analysis, 2th ed. p. 679, McGraw-Hill, 1991.

6. Marshall, K.T. and Oliver, R.M. , Decision Making and Forecasting, p. 83-130, McGraw-Hill, 1995.

7. Gaver, D. P. and Jacobs, P. A. " Sequential Tasks and Information (A Prototypical Combat Information War Vignette)", Working Paper, Operations Research Department, Naval Postgraduate School, Monterey, CA, 1996.

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